AP Calculus AB
Area Between Curves Notes

Name Key

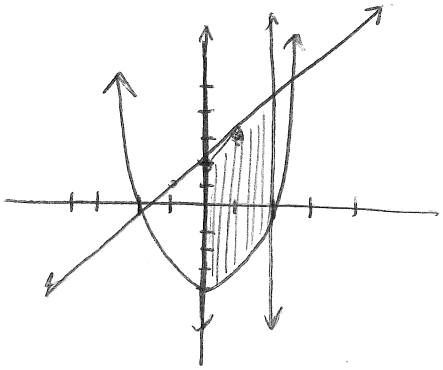
$$\text{Area} = \int_a^b [f(x) - g(x)]\,dx$$

Where f and g are continuous functions and $f(x) \geq g(x)$ for all x in [a, b]

Examples:

Find the area between the curves.

1. $y = x^2 - 4$, $y = x + 2$, x = 0, and x = 2



$$\int_0^2 x+2-(x^2-4)\,dx$$

$$\int_0^2 -x^2+x+6\,dx$$

$$-\frac{1}{3}x^3+\frac{1}{2}x^2+6x\Big|_0^2$$

$$\left[-\frac{1}{3}(2)^3+\frac{1}{2}(2)^2+6(2)\right]-[0]$$

$$-\frac{8}{3}+2+12$$

$$-\frac{8}{3}+14$$

$$\frac{34}{3}\ \text{units}^2$$

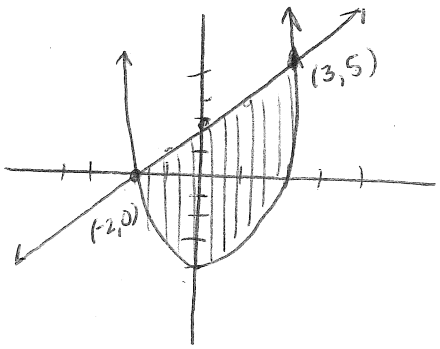
2. $y = x^2 - 4$, $y = x + 2$

$x^2-4 = x+2$

$x^2-x-6=0$

$(x-3)(x+2)=0$

$x=3 \quad x=-2$



$$\int_{-2}^3 x+2-(x^2-4)\,dx$$

$$\int_{-2}^3 -x^2+x+6\,dx$$

$$-\frac{1}{3}x^3+\frac{1}{2}x^2+6x\Big|_{-2}^3$$

$$\left[-\frac{1}{3}(3)^3+\frac{1}{2}(3)^2+6(3)\right]-\left[-\frac{1}{3}(-2)^3+\frac{1}{2}(-2)^2+6(-2)\right]$$

$$\left[-9+\frac{9}{2}+18\right]+\left[\frac{8}{3}+2+12\right]$$

$$19+\frac{9}{2}-\frac{8}{3}$$

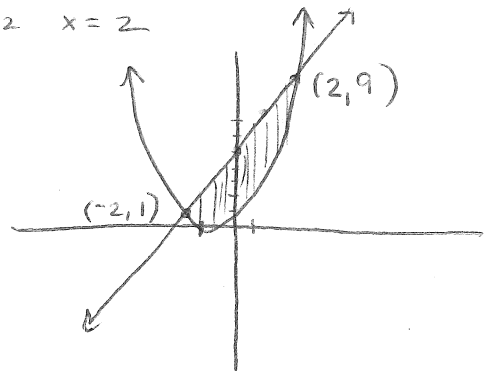
3. $f(x)=x^2+2x+1$ and $g(x)=2x+5$

$x^2+2x+1 = 2x+5$

$x^2-4=0$

$(x+2)(x-2)=0$

$x=-2 \quad x=2$



$$\int_{-2}^{2} 2x+5-(x^2+2x+1)\,dx$$

$$\int_{-2}^{2} 2x+5-x^2-2x+1\,dx$$

$$\int_{-2}^{2} -x^2+4\,dx$$

$$-\frac{1}{3}x^3+4x\Big|_{-2}^{2}$$

$$\left[-\frac{1}{3}(2)^3+4(2)\right]-\left[-\frac{1}{3}(2)^3+4(-2)\right]$$

$$-\frac{8}{3}+8-\frac{8}{3}+8$$

$$-\frac{16}{3}+16$$

$$\frac{32}{3}$$

4. $y=x^3-4x$ and $y=5x$

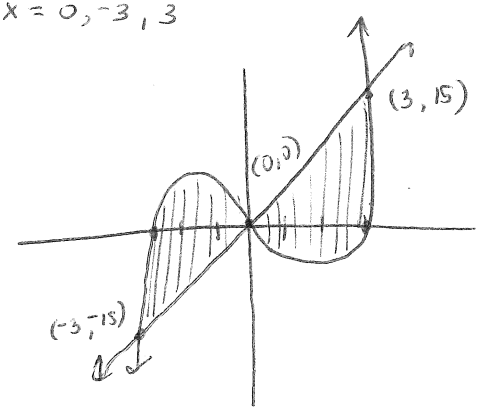
$x^3-4x=5x$

$x^3-9x=0$

$x(x^2-9)=0$

$x(x+3)(x-3)=0$

$x=0,-3,3$



$$\int_{-3}^{0} x^3-4x-5x\,dx+\int_{0}^{3} 5x-(x^3-4x)\,dx$$

$$\int_{-3}^{0} x^3-9x\,dx+\int_{0}^{3} -x^3+9x\,dx$$

$$\frac{1}{4}x^4-\frac{9}{2}x^2\Big|_{-3}^{0}+-\frac{1}{4}x^4+\frac{9}{2}x^2\Big|_{0}^{3}$$

$$[0]-\left[\frac{1}{4}(-3)^4-\frac{9}{2}(-3)^2\right]+\left[-\frac{1}{4}(3)^4+\frac{9}{2}(3)^2\right]-[0]$$

$$-\frac{81}{4}+\frac{81}{2}+\frac{-81}{4}+\frac{81}{2}$$

5. $x+y^2=2$ and $x+y=0$

$y^2=2-x \qquad y=-x$

$y=\pm\sqrt{2-x}$

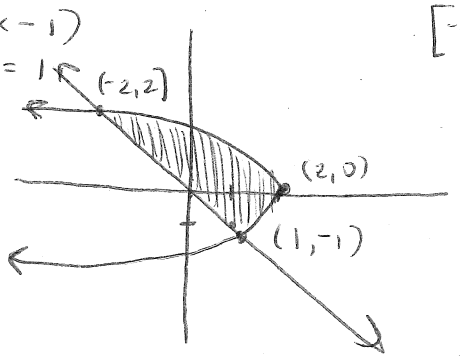
$(\pm\sqrt{2-x})^2=(-x)^2$

$2-x=x^2$

$0=x^2+x-2$

$0=(x+2)(x-1)$

$x=-2 \quad x=1$



$$\int_{-1}^{2} 2-y^2-(-y)\,dy$$

$$\int_{-1}^{2} -y^2+y+2\,dy$$

$$-\frac{1}{3}y^3+\frac{1}{2}y^2+2y\Big|_{-1}^{2}$$

$$\left[-\frac{1}{3}(2)^3+\frac{1}{2}(2)^2+2(2)\right]-\left[-\frac{1}{3}(-1)^3+\frac{1}{2}(-1)^2+2(-1)\right]$$

$$\left[-\frac{8}{3}+2+4\right]+\left[+\frac{1}{3}+\frac{1}{2}-2\right]$$

$$-\frac{3}{3}+4-\frac{1}{2}$$